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# Optimum reserve size, fishing induced carrying capacity change and phenotypic diversity 

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# Optimum reserve size, fishing induced carrying capacity change and phenotypic diversity 


#### Abstract

In spite of seemingly good effort limiting fishing policies in several parts of the world, fish stock have been heavily overexploited with some stocks completely collapsed or at the verge of doing so. Recent studies have found complexity of ecological systems, interdependency among species, and negative impact of fishing activities on environmental carrying capacity of fish stocks as potential contributors. A number of biologists, managers and practitioners strongly support the use of marine reserve as a management strategy for marine conservation. This paper contributes to this line of research by seeking an optimum reserve size and fishing effort for situations where fishing activities destroy habitats as well as negatively impact species diversity. We found that a reserve size which maximizes economic rents could collapse fish stocks if fishing impacts are not accounted for. On the other hand, the reserve serves as a burfication parameter that could improve the resilience of marine ecosystems.


Keywords: marine reserve, fishing impact on carrying capacity, fishing policy, phenotypic diversity, stock collapse

JEL Classification: C61, Q22, Q57, Q58

| Nome |  |
| :---: | :---: |
| Letter | Definition |
| $x$ | Fish stock (in biomass e.g., tons of fish in the aquatic environment) |
| E | Fishing Effort (e.g., \# of days of fishing) |
| $h$ | Catch/Harvest quantity (in biomass) |
| s | Average Phenotype (i.e., a measure of the average of phenotypic characteristics in a population) |
| $v$ | Phenotypic variance (i.e., a measure of diversity, represents the spread of individual species' phenotypes around the mean) |
| $\delta$ | Social discount rate |
| $\sigma$ | Catchability coefficient (a measure of gear efficiency) |
| c | Cost per unit effort |
| m | Proportion of management area that is reserved (i.e., not fished) |
| $p$ | A competitive price per quantity measure (e.g., kilogram) of fish |
| k | Environmental carrying capacity of the fish stock |

## 1. Introduction

Fishery resources in many part of the world are heavily overexploited with some stocks completely collapsed or at the verge of doing so (see e.g., Myers and Worm, 2003; Sumaila and Cheung, 2009). This phenomenon has also been acknowledged in an FAO publication which noted 69 percent of the world's marine stocks are either fully to heavily exploited, overexploited, or depleted (Lauck et al., 1998). In a bid to control the over-fishing problem, countries have advanced policies including moratoria and restrictions on fishing methods. Unfortunately, the outcomes have been quite disappointing as some fisheries such as the North Sea cod have collapsed in spite of seemingly good policies (European Environmental Agency, 2003). A number of emerging studies have attributed the unanticipated overexploitation and collapse to inadequate accounting for species diversity (Sterner, 2007; Akpalu, 2009; Akpalu and Bitew, 2011) and fishing impact on habitats and production of planktons (Armstrong, 2007; Armstrong and Falk-Petersen, 2008; Akpalu and Bitew, 2011).

In recent years there has been strong campaign by biologists, economists, managers, and practitioners for the use of marine reserve as a management strategy for marine conservation (Man et al., 1994; Allison et al., 1998; Boersma and Parrish 1999; Pezzey et al., 2000; Sanchirico and wilen, 2001; Armstrong and Skonhoft, 2006). This involves protection of critical areas from some or all human activities (Bergen and Carr, 2003) in order to conserve species and communities (Allison et al 1998). Some economists and fishers believe reserves are less effective or beneficial than other traditional tools, especially if populations are not overexploited (Holland and Brazee, 1996; Bergen and Carr, 2003), but scientists present the contrary argument that reserves generate ecosystem benefits within and outside its boundaries (see e.g., Gell and

Roberts, 2003) and improve the resilience of ecosystems (see e.g., Apostolaki, et al., 2002; Grafton et al., 2009). Worm et al. (2006) noted that large proportions of overfished stocks impact biodiversity and alter ecosystem functioning. An important question which has not been addressed adequately in the bio-economics literature is whether reserves could guarantee resilient ecosystems if fishing impact on species diversity and alteration of habitat is accounted for in calculating catch potentials. This paper seeks to fill this gap and found that (1) if the reserve size surpasses a threshold, the fisheries ecosystem is resilient and the stock may not collapse even if an effort-limiting policy does not account for the negative impacts of fishing activities. Conversely if the reserve is below the threshold the fishery could collapse if the negative fishing impact is not accounted for; (2) if fishing impacts habitat, all else being equal, the equilibrium fishing effort and reserve size that maximize economic surplus have to be set lower than otherwise; (3) if fishing degrades or lowers species diversity, all other things being equal, the equilibrium reserve size must increase but equilibrium fishing effort must be reduced; and (4) if future benefits and costs are discounted at a positive discount rate, 1 through 3 applies but in each case the equilibrium effort and reserve size must be set lower than in a zero discount or static scenario.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 presents the static model and presents numerical results. Section 4 contains the dynamic version of the model and empirically illustrates the results. Section 5, which is the final section, concludes the paper.

## 2. The Theoretical Model

In the following sections (i.e., section 3 and 4) static and dynamic models are presented and conditions for optimal reserve sizes and fishing effort have been derived. For each case three scenarios are discussed: a basic situation where the impact of fishing on the environmental carrying capacity of the stock and phenotypic diversity of the stock are ignored; a second situation where fishing impact on environmental carrying capacity is considered but its effect on phenotypic diversity is not accounted for; and the third scenario where both effects are considered. As noted by Armstrong and Skonhoft (2006) ecological conditions outside a reserve naturally differ from those within the reserve due to fishing pressure. We begin with the static model followed by the dynamic analysis.

## 3. Optimum Reserve Size in a Timeless (Static) Framework

Suppose the equations of motion defining the stock of fish within and outside of a marine reserve are given by

$$
\begin{gather*}
\dot{x}=g(x, 1-m)+d \frac{y}{m}-h,  \tag{1}\\
\dot{y}=f(y, m)-d \frac{y}{m}, \tag{2}
\end{gather*}
$$

where m is the reserve size, $x=\frac{X}{K}$ and $y=\frac{Y}{K}$ are stocks of fish in the fishing and reserve areas (i.e., $X$ and $Y$ ) respectively, scaled down by the total environmental carriage capacity ( $K$ ); $h$ is harvest; and dot (.) is time derivative. Like Sanchirico and Wilen (2001) and Conrad (1999) we
have assumed symmetric density dependent stocks. Suppose the growth functions are logistic (i.e., $f(y, m)=r y\left(1-\frac{y}{m}\right)$, and $\left.g(x, 1-m)=r x\left(1-\frac{x}{1-m}\right)\right)$ and the harvest function is of the Schaefertype (i.e., $h=\sigma E x$, where $E$ is fishing effort and $\sigma$ is catchability coefficient, a measure of gear effectiveness). In steady state $\dot{y}=\dot{x}=0$ implying, from equation (2), $y=m\left(1-d(m r)^{-1}\right)$. In addition, from equation (1), we have:

$$
\begin{equation*}
x^{*}(E, m)=\frac{m(1-m)(r-\sigma E)+\sqrt{4 m d(m-1)(d-m r)+(m(1-m)(r-\sigma E))^{2}}}{2 m r} \tag{3}
\end{equation*}
$$

The corresponding yield is

$$
\begin{equation*}
Y=Y^{*}(E, m)=\sigma E\left(\frac{m(1-m)(r-\sigma E)+\sqrt{4 m d(m-1)(d-m r)+(m(1-m)(r-\sigma E))^{2}}}{2 m r}\right) \tag{4}
\end{equation*}
$$

Furthermore let the instantaneous profit function of the social planner be

$$
\begin{equation*}
\pi(E, m)=p Y^{*}(E, m)-c E \tag{5}
\end{equation*}
$$

Fig. 1 presents the plot of the first and second terms on the right hand side of the profit function. The thick and broken concave functions are the revenue function for relatively low and high values of the reserve size, which were arbitrarily chosen. The straight upward sloping line is the cost function. Any vertical gap between the concave and the straight line curves measure the
instantaneous profit. Consequently, the largest gap between the two functions determines the level of effort that maximizes profit. From the two concave functions, from analytical point of view, increasing the reserve size must be accompanied by lowering the optimum effort level in order to obtain maximum economic surplus from the fishery (i.e., $E_{1}>E_{2}$ ). It is noteworthy that the reserve size cannot be chosen arbitrarily but optimally and derived from equation 5 .


Fig. 1. Optimal fishing effort if marine reserve size is low or high and fishing impact on carrying capacity and phenotypic diversity is ignored. Note that $m_{2}>m_{1}$.

The first order condition of equation (5) with respect to E and m are as follows:

$$
\begin{align*}
& \frac{\partial \pi(E, m)}{\partial E}=p Y_{E}^{*}(E, m)-c=0  \tag{6}\\
& \frac{\partial \pi(E, m)}{\partial m}=0 \Rightarrow Y_{m}^{*}(E, m)=0 \tag{7}
\end{align*}
$$

Equation (6) stipulates that, in equilibrium, the value of the marginal productivity of effort (i.e., $p Y_{E}^{*}$ ) must equate the marginal cost of effort (i.e., c). According to equation (7), in order to maximize economic surplus from the fishery, the social planner must set the reserve size to a level that equates the marginal net benefit from the chosen reserve size to zero. Solving the two equations simultaneously yields $E^{*}=E(p, c ; \sigma, d, r)$ and $m^{*}=m(p, c ; \sigma, d, r)$. Due to the complicated nature of the functional form the explicit parametric solutions are long, complicated and devoid of any direct economics intuition hence the optimal values calculated (see Table 5) based on some chosen parameter values (see Table 4).

## Fishing Negatively Impact Carrying Capacity

To extend the model a step further, suppose fishing negatively impact carrying capacity. Let this impact be linearly increasing in fishing effort, i.e., $\varepsilon E$, where $\varepsilon(0,1)$ is a constant. The stock evolution within the fishing area becomes

$$
\begin{equation*}
\dot{x}=r x\left(1-\frac{x}{1-m-\varepsilon E}\right)+d \frac{y}{m}-h, \tag{8}
\end{equation*}
$$

If the negative externality does not extend to the reserve area, following the routine prior calculations, the corresponding steady state stock level similar to equation (3) is

$$
x^{* *}(E, m)=\frac{m(m+\varepsilon E-1)(\sigma E-r)+\sqrt{m(m+\varepsilon E-1)\left(4 d^{2}-4 d m r+m(m+\varepsilon E-1)\right)(r-\sigma E)^{2}}}{2 m r}(9)
$$

The profit function corresponding to equation (5) is

$$
\begin{equation*}
\pi(E, m)=p Y^{* *}(E, m)-c E, \tag{10}
\end{equation*}
$$

where $Y^{* *}(E, m)=\sigma E x^{* *}(E, m)$ is the yield function.

Fig. 2 compares the plots of the preceding situation (where fishing does not impact carrying capacity, i.e., $A(m)$ ) and a situation where it does (i.e., $B(m)$ ). The broken concave functions (i.e., $B(m)$ ) represent the latter case. Clearly, for any given $m$, the optimum effort must be lower if the fishing negatively impacts carrying capacity. In addition, in each of the two cases, higher values of $m$ results in lower equilibrium effort.


Fig. 2. Optimal fishing effort if marine reserve size is low or high if fishing impact carrying capacity but phenotypic diversity is ignored. Note that $m_{2}>m_{1}$.

Fishing negatively impact carrying capacity; and phenotypic diversity in the fishing area

Empirical studies have found that marine reserves significantly improve species diversity (see e.g., Halpern, 2003). To further align our model with realism, suppose there is no significant gain in diversity within the marine reserve but intense fishing erodes diversity within the fishing area. To account for the biodiversity (following Norberg et al., 2001; Akpalu, 2009; and Akpalu and Bitew, 2011), let the dynamic equation of the stock be re-specified as

$$
\begin{equation*}
\dot{x}=r x\left(1-\frac{x}{1-m-\varepsilon E}\right)-\frac{\alpha}{s^{3}}\left(s^{2}+2 v\right) x+d\left(1-\frac{d}{m r}\right)-\sigma E x . \tag{11}
\end{equation*}
$$

where $s$ is the average phenotype of the functional group under consideration, $v$ is phenotypic variance which is assumed to be constant. The model uses moment approximation methods to capture the dynamics of the macroscopic or aggregate characteristics of a functional group of species in terms of total biomass, average phenotype, and phenotypic variance. In addition, since we have assumed the growth function is logistic, it has a global maximum. Also if there are large varieties of species of which some underperform (i.e., if $v$ is high), the average growth rate of the total biomass will be low. Following Akpalu (2009) and Akpalu and Bitew (2011) define the equation of motion of the average phenotype as

$$
\begin{equation*}
\dot{s}=v f_{s}-\frac{\alpha s}{x} . \tag{12}
\end{equation*}
$$

In steady state, $\dot{x}=0$ and $s=(v x)^{1 / 3}$, we have

$$
\begin{equation*}
r x^{\prime}\left(1-\frac{x^{\prime}}{1-m-\varepsilon E}\right)=\alpha\left(v^{-1 / 3} x^{\prime 2 / 3}+2\right)+\sigma x^{\prime} E-d\left(1-\frac{d}{m r}\right), \tag{13}
\end{equation*}
$$

where $x^{\prime}=x^{\prime}(E, m)$ is obtained by solving for the optimum stock from equation (11). The related profit function is

$$
\begin{equation*}
\pi(E, m)=p Y^{\prime}(E, m)-c E \tag{14}
\end{equation*}
$$

where $Y^{\prime}(E, m)=\sigma E x^{\prime}(E, m)$ is the yield function.

A plot of equation (14) is presented in Fig. 3. From the figure, relatively low values of the reserve size generate critical depensation in the yield function implying effort levels beyond a given threshold will collapse the stock. This function is represented by the function with the combination of thick and broken lines (i.e., $R\left(m_{1}\right)$ ). However if $m$ is large enough, the yield function indicates noncritical depensation meaning the stock within the fishing area is resilient to high effort levels (i.e., $R\left(m_{2}\right)$ ). Thus the reserve size is a burfication parameter that could preserve or collapse the stock.


Fig. 3. Optimal fishing effort if marine reserve size is low or high if fishing impact carrying capacity and phenotypic diversity is accounted for. Note that $m_{2}>m_{1}$.

Comparing the three profit functions it is evident that optimum effort levels could be set higher if both fishing impact on carrying capacity and phenotypic diversity within the fishing area are ignored. The plots of the three cases are presented in Fig. 4. The compensated yield function (i.e., $A\left(m_{1}\right)$ ) is the baseline situation which yields the highest optimum effort for any given $m$. On the other hand, the critical depensation function (i.e., $R\left(m_{1}\right)$ ) generates the lowest optimum effort. If the reserve is relatively small, an effort limiting policy may collapse the stock. However if the reserve size is set high enough (i.e., say $m_{2}$ so that the corresponding function is $R\left(m_{2}\right)$ ) the equilibrium effort may generate lower economic surplus but will not collapse the stock.


Fig. 4. Optimal fishing effort if marine reserve size is low or high under the three scenarios. Note that $m_{2}>m_{1}$.

## Numerical Illustration

To empirically illustrate the optimum values of the decision variables, including $m$, we employed the parameter values in Table 4. The corresponding results presented in Table 5 reveals the following:

1. For any given phenotypic diversity, the optimum reserve and fishing effort must be set lower if the impact of fishing on carrying capacity becomes intense.
2. For any given fishing impact on carrying capacity, lower diversity must be accompanied by bigger reserve size but lower fishing effort and harvest.
3. Greater fishing impact of environmental carrying capacity and lower diversity necessitates decreasing the reserve size and lowering catch or fishing effort levels.

Clearly, ignoring negative fishing impact on carrying capacity and diversity could result in overfishing and possible stock collapse.

Table 5. Optimum reserve, effort and harvest if fishing impact capacity and species diversity in a timeless setting

| Carrying capacity <br> impact of catch | Phenotypic <br> variance | Reserve size | Optimum <br> effort | Optimum <br> stock | Optimum <br> harvest |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\varepsilon$ | $v$ | $m$ | $E^{*}$ | $x^{*}$ | $h^{*}$ |
| $\mathbf{0 . 0 0 0}$ | $\mathbf{0 . 0 0}$ | $\mathbf{0 . 1 2 5 7 8 6}$ | $\mathbf{5 . 7 8 1 2 4}$ | $\mathbf{0 . 4 3 7 1 0 7}$ | $\mathbf{0 . 3 9 7 5}$ |
| 0.001 | 0.00 | 0.125369 | 5.76404 | 0.435879 | 0.3952 |
| 0.010 | 0.00 | 0.121507 | 5.59636 | 0.425748 | 0.3748 |
| 0.001 | 0.01 | 0.140599 | 3.55029 | 0.382842 | 0.2138 |
| $\mathbf{0 . 0 1 0}$ | $\mathbf{0 . 0 1}$ | $\mathbf{0 . 1 3 8 0 4 9}$ | $\mathbf{3 . 4 3 8 2 0}$ | $\mathbf{0 . 3 7 7 0 2 8}$ | $\mathbf{0 . 2 0 3 9}$ |
| 0.100 | 0.01 | 0.115952 | 2.33770 | 0.352715 | 0.1297 |
| 0.001 | 0.10 | 0.131890 | 4.12457 | 0.411999 | 0.2673 |
| $\mathbf{0 . 0 1 0}$ | $\mathbf{0 . 1 0}$ | $\mathbf{0 . 1 2 9 0 6 9}$ | $\mathbf{3 . 9 9 1 0 8}$ | $\mathbf{0 . 4 0 4 9 4 0}$ | $\mathbf{0 . 2 5 4 2}$ |

## 4. A Dynamic model of optimum reserve Size

This section provides a dynamic version of the problem discussed earlier. The dynamic version makes it possible to include discounting in the analysis. Suppose the catch is sold in a competitive market and the unit price is $p$ and the cost of harvest is $c E$. The instantaneous profit function is $p \sigma E x-c E$. If all future costs and benefits are discounted at a positive social discount rate of $\delta$, the social planner's optimization problem can be stated as:

$$
\begin{equation*}
V(x, s, E)=\operatorname{Max}_{E} \int_{0}^{\infty}(p \sigma x-c) E e^{-\delta t} d t \tag{15}
\end{equation*}
$$

subject to equations (16) and (17). That is,

$$
\begin{gather*}
\dot{x}=r x\left(1-\frac{x}{1-m-\varepsilon E}\right)-\frac{\alpha}{s^{3}}\left(s^{2}+2 v\right) x+d\left(1-\frac{d}{m r}\right)-\sigma E x .  \tag{16}\\
\dot{s}=v f_{s}-\frac{\alpha s}{x} . \tag{17}
\end{gather*}
$$

The corresponding current-value Hamiltonian of the program is specified as:
$H(\cdot)=(\sigma p x-c) E+\lambda\left(r x\left(1-\frac{x}{1-m-\varepsilon E}\right)-\frac{\alpha}{s^{3}}\left(s^{2}+2 v\right) x+d\left(1-\frac{d}{m r}\right)-\sigma E x\right)+\mu\left(\frac{v \alpha}{s^{2}}-\frac{\alpha s}{x}\right)$,
where $\lambda$ and $\mu$ are the shadow or scarcity values of the stock within the fishing area, and the average phenotype respectively. From the maximum principle, the first-order conditions with respect to the flow variables (i.e., fishing effort and reserve size) are denoted by:

$$
\begin{gather*}
\frac{\partial H(\cdot)}{\partial E}=p \sigma x-c-\lambda\left(\frac{r \varepsilon x^{2}}{(1-m-\varepsilon E)^{2}}+\sigma x\right)=0 .  \tag{19}\\
\frac{\partial H(\cdot)}{\partial m}=\lambda\left(r^{-1}\left(\frac{d}{m}\right)^{2}-\frac{r x^{2}}{(1-m-\varepsilon E)^{2}}\right)=0 . \tag{20}
\end{gather*}
$$

The corresponding costate equations are expressed as:

$$
\begin{gather*}
\dot{\lambda}-\delta \lambda=-\frac{\partial H(\cdot)}{\partial x}=-\sigma p E-\lambda\left(r\left(1-\frac{2 x}{(1-m-\varepsilon E)}\right)-\frac{\alpha}{s^{3}}\left(s^{2}+2 v\right)-\sigma E\right)-\mu \frac{\alpha s}{x^{2}}  \tag{21}\\
\dot{\mu}-\delta \mu=-\frac{\partial H(\cdot)}{\partial s}=\mu\left(\frac{2 \alpha v}{s^{3}}+\frac{\alpha}{x}\right)-\lambda \alpha x\left(\frac{1}{s^{2}}+\frac{6 v}{s^{4}}\right) \tag{22}
\end{gather*}
$$

From the maximum principle, in inter-temporal equilibrium, profit per unit of effort (i.e., $p \sigma x-c$ ) must reflect some upwards adjusted shadow value of the stock (i.e., $\lambda \sigma x\left(\frac{r \varepsilon x}{(1-m-\varepsilon E)^{2} \sigma}+1\right)$ ). The scaling term $\frac{r \varepsilon x}{(1-m-\varepsilon E)^{2} \sigma}$ which results from the negative impact of fishing on carrying capacity guarantees a lower catch trajectory. Secondly, from equation (20) in inter-temporal equilibrium, the marginal benefit from increasing the reserve size (i.e., $r^{-1}\left(\frac{d}{m}\right)^{2}$ ) must reflect the marginal opportunity cost of doing so (i.e., $r\left(\frac{x}{1-m-\varepsilon E}\right)^{2}$ ).

From the first costate equation (i.e., equation 19), in dynamic equilibrium, the sum of the capital gain from preserving the fish stock and some stock effect (i.e., $\dot{\lambda}+\sigma p E+\lambda\left(r\left(1-\frac{2 x}{(1-m-\varepsilon E)}\right)-\frac{\alpha}{s^{3}}\left(s^{2}+2 v\right)-\sigma E\right)+\mu \frac{\alpha s}{x^{2}}$ ) must balance the interest earnable on the shadow value of the stock if it is harvested now and the proceed deposited in a bank (i.e., $\delta \lambda$ ). Equation (20) stipulates that, in a dynamic equilibrium, the capital gain on the shadow value of improved diversity plus some stock effect (i.e., $\dot{\mu}+\lambda \alpha x\left(\frac{1}{s^{2}}+\frac{6 v}{s^{4}}\right)-\mu\left(\frac{2 \alpha v}{s^{3}}+\frac{\alpha}{x}\right)$ ) must equate
the opportunity cost of marginally preserving the diversity (i.e., $\delta \mu$ ). In steady $\dot{s}=\dot{\mu}=\dot{\lambda}=\dot{x}=0$, implying the following:

$$
\begin{gather*}
p \sigma x-c=\lambda\left(\frac{r \varepsilon x^{2}}{(1-m-\varepsilon E)^{2}}+\sigma x\right) .  \tag{23}\\
r x m=d(1-m-\varepsilon E) .  \tag{24}\\
r x\left(1-\frac{x}{1-m-\varepsilon E}\right)-\frac{\alpha}{s^{3}}\left(s^{2}+2 v\right) x+d\left(1-\frac{d}{m r}\right)=\sigma E x .  \tag{25}\\
v \lambda=s^{3} .  \tag{26}\\
\delta \lambda=\sigma p E+\lambda\left(r\left(1-\frac{2 x}{(1-m-\varepsilon E)}\right)-\frac{\alpha}{s^{3}}\left(s^{2}+2 v\right)-\sigma E\right)+\mu \frac{\alpha s}{x^{2}} .  \tag{27}\\
\delta \mu \alpha x\left(\frac{1}{s^{2}}+\frac{6 v}{s^{4}}\right)-\mu\left(\frac{2 \alpha v}{s^{3}}+\frac{\alpha}{x}\right) . \tag{28}
\end{gather*}
$$

The preceding results are compared to situations where (1) fishing impact on carrying capacity and diversity are ignored; and (2) fishing impacts carrying capacity but phenotypic diversity is not accounted for. Regarding the first situation we have $\dot{s}=s=\varepsilon=0$ and the equation of motion becomes $\quad \dot{x}=\rho x\left(1-\frac{x}{1-m}\right)+d\left(1-\frac{d}{m r}\right)-\sigma E x$. The corresponding revised current value Hamiltonian is

$$
\begin{equation*}
H(.)=(\sigma p x-c) E+\omega\left(r x\left(1-\frac{x}{1-m}\right)+d\left(1-\frac{d}{m r}\right)-\sigma E x\right) . \tag{29}
\end{equation*}
$$

From the maximum principle, the first order conditions are

$$
\begin{gather*}
\frac{\partial H(\cdot)}{\partial E}=p \sigma x-c-\omega \sigma x=0 .  \tag{30}\\
\frac{\partial H(\cdot)}{\partial m}=\omega\left(\frac{d^{2}}{m^{2} r}-\frac{r x^{2}}{(1-m)^{2}}\right)=0 . \tag{31}
\end{gather*}
$$

The costate equation is

$$
\begin{equation*}
\dot{\omega}-\delta \omega=-\frac{\partial H(\cdot)}{\partial x}=-\sigma p E-\omega\left(r\left(1-\frac{2 x}{1-m}\right)-\sigma E\right) . \tag{32}
\end{equation*}
$$

In steady state $\dot{\omega}=\dot{x}=0$, therefore $x=d(1-m)(m r)^{-1}$ and

$$
\begin{gather*}
(d(1-m))(m r-d)+d(m r-d) m=\sigma d(1-m) m E .  \tag{33}\\
\delta\left(p-\frac{c m r}{\sigma d(1-m)}\right)=\sigma p E+\left(p-\frac{c m r}{\sigma d(1-m)}\right)\left(\left(1-\frac{2 d}{m r}\right) r-\sigma E\right) . \tag{34}
\end{gather*}
$$

For the second situation where fishing impact on environmental carrying capacity but diversity is ignored, the corresponding current-value Hamiltonian of the program is

$$
\begin{equation*}
H(.)=(\sigma p x-c) E+\varphi\left(r x\left(1-\frac{x}{1-m-\varepsilon E}\right)+d\left(1-\frac{d}{m r}\right)-\sigma E x\right) . \tag{35}
\end{equation*}
$$

$$
\begin{gather*}
\frac{\partial H(\cdot)}{\partial E}=p \sigma x-c-\varphi\left(\frac{r \varepsilon x^{2}}{(1-m-\varepsilon E)^{2}}+\sigma x\right)=0 .  \tag{36}\\
\frac{\partial H(\cdot)}{\partial m}=0 \Rightarrow \frac{d}{m}=\frac{r x}{(1-m-\varepsilon E)} . \tag{37}
\end{gather*}
$$

The costate equation is

$$
\begin{equation*}
\dot{\varphi}-\delta \varphi=-\frac{\partial H(\cdot)}{\partial x}=-\sigma p E-\varphi\left(r\left(1-\frac{2 x}{1-m-\varepsilon E}\right)-\sigma E\right) . \tag{38}
\end{equation*}
$$

In steady state we have $x=d(1-m-\varepsilon E)(m r)^{-1}$ and

$$
\begin{gather*}
\delta \varphi=\sigma p E+\varphi\left(r\left(1-\frac{2 x}{1-m-\varepsilon E}\right)-\sigma E\right) .  \tag{39}\\
p \sigma x-c=\varphi\left(\frac{r \varepsilon x^{2}}{(1-m-\varepsilon E)^{2}}+\sigma x\right) .  \tag{40}\\
\rho x\left(1-\frac{x}{1-m}\right)+d\left(1-\frac{d}{m r}\right)=\sigma E x . \tag{41}
\end{gather*}
$$

The parametric solutions are very complicated and difficult to compute and compare hence we resort to numerical solutions using the parameter values reported in Table 4. The results reported in Table 6 shows lower optimum reserve size in steady state compared to the static scenario presented in the preceding section. For example, for the benchmark case where fishing impact on carrying capacity and phenotypic diversity is ignored, the $5 \%$ discount rate lowers the optimum reserve size
and effort by $28 \%$ and $42 \%$ respectively. Generally the trend holds for all levels of fishing impact on carrying capacity and phenotypic diversity. Moreover intense fishing impact on the carrying capacity requires lowering the reserve size and reducing fishing effort levels. Also, as found in the static case, declining species diversity requires bigger reserve size but lower fishing effort.

Table 6. Optimum reserve, effort, and harvest if fishing impact capacity and species diversity in a dynamic modeling setting.

| Carrying capacity <br> impact of catch | Phenotypic <br> variance | Reserve size | Optimum <br> effort | Optimum <br> stock | Optimum <br> harvest |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\varepsilon$ | $v$ | $m$ | $E^{*}$ | $x^{*}$ | $h^{*}$ |
| $\mathbf{0 . 0 0}$ | $\mathbf{0 . 0 0}$ | $\mathbf{0 . 0 9 0 2 1 0 4}$ | $\mathbf{3 . 3 6 4 4 1}$ | $\mathbf{0 . 6 3 4 2 8 9}$ | $\mathbf{0 . 3 3 5 7}$ |
| 0.01 | 0.00 | 0.0878258 | 3.15575 | 0.630620 | 0.3130 |
| 0.10 | 0.00 | 0.0750370 | 1.80067 | 0.624343 | 0.1768 |
| $\mathbf{0 . 0 0}$ | $\mathbf{0 . 0 1}$ | $\mathbf{0 . 0 9 3 8 5 2 9}$ | $\mathbf{1 . 4 4 8 3 9}$ | $\mathbf{0 . 6 0 7 2 3 1}$ | $\mathbf{0 . 1 3 8 3}$ |
| 0.01 | 0.01 | 0.0932112 | 1.38933 | 0.602470 | 0.1317 |
| 0.10 | 0.01 | 0.0887971 | .970345 | 0.576658 | 0.0880 |
| $\mathbf{0 . 0 0}$ | $\mathbf{0 . 1 0}$ | $\mathbf{0 . 0 8 7 3 4 4 3}$ | $\mathbf{1 . 6 9 6 4 2}$ | $\mathbf{0 . 6 5 7 1 6 6}$ | $\mathbf{0 . 1 7 5 4}$ |
| 0.01 | 0.10 | 0.0866695 | 1.62749 | 0.650963 | 0.1666 |

## 5. Conclusion

Recent studies have found that fishing activities impact species diversity as well as environmental carrying capacity of stocks within fishing areas. We have shown that both marine reserve sizes and fishing potentials are overestimated and stocks could potentially collapse if these fishing impacts are
neglected. Conversely, if the reserve size is high enough, net economic surplus levels will be low but the stock will not collapse if the fishing impacts are not accounted for, implying the reserve size is a burfication parameter that could guarantee the resilience of the fishery. Furthermore, all else being equal, the optimum reserve and fishing effort must be set lower if the impact of fishing on carrying capacity becomes intense; and the reserve size should be set bigger but fishing effort set lower if fishing degrades diversity. As a policy advice in order to prevent stock collapse reserves sizes must be set high enough especially in situations where fishing impacts are difficult to ascertain.

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## Appendix 1

Table 4. Parameter values used for simulations

| Parameter | Description | Value |
| :---: | :--- | :--- |
| $\rho$ | Intrinsic growth rate * | 1.59 |
| $\varepsilon$ | Phenotypic variance | $0.01,0.10$ |
| $d$ | Severity of fishing impact on carrying capacity | $0.001,0.01,0.10$ |
| $\alpha$ | Dispersion parameter | 0.1 |
| $k$ | A scalar | 0.4 |
| $\sigma$ | Carrying capacity* | 253910 tons |
| $\delta$ | Catchability coefficient | 0.01573 |
| $c$ | Social discount rate* | 0.05 |
| $p$ | Cost per unit effort | 0.56 |

The figure associated with the starred parameter is taken from Akpalu and worku (2011). Some of the figures are averages for the three tuna species harvested in Ghana.

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